

# Cost-Benefit Analysis of Two-Identical Warm Standby Solar- and Fuel-Cell-Aircraft System Failure due to Reduction in Solar Power and Persistent, High Dihedral Configuration

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**Abstract**—The NASA Helios Prototype was the fourth and final aircraft developed as part of an evolutionary series of solar- and fuel-cell-system-powered unmanned aerial vehicles. AeroVironment, Inc. developed the vehicles under NASA's Environmental Research Aircraft and Sensor Technology (ERAST) program. They were built to develop the technologies that would allow long-term, high-altitude aircraft to serve as "atmospheric satellites", to perform atmospheric research tasks as well as serve as communications platforms. In the present paper we have taken failure due to reduction in solar power and persistent, high dihedral configuration with different repair facilities. When the main unit fails then warm standby system becomes operative. Failure due to reduction in solar power cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III or Type IV repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

## 1. INTRODUCTION

The NASA Centurion was modified into the Helios Prototype configuration by adding a sixth 41 feet (12 m) wing section and a fifth landing gear and systems pod, becoming the fourth configuration in the series of solar-powered flying wing demonstrator aircraft developed by AeroVironment under the ERAST project. The larger wing on the Helios Prototype accommodated more solar arrays to provide adequate power for the sun-powered development flights that followed. The aircraft's maiden flight was on September 8, 1999.

The ERAST program had two goals when developing the Helios Prototype: 1) sustained flight at altitudes near 100,000 feet (30,000 m) and 2) endurance of at least 24 hours, including at least 14 of those hours above 50,000 feet (15,000 m). To this end, the Helios Prototype could be configured in two different ways. The first, designated HP01, focused on achieving the altitude goals and powered the aircraft with batteries and solar cells. The second configuration, HP03, optimized the aircraft for endurance, and used a combination

of solar cells, storage batteries and a modified commercial hydrogen-air fuel cell system for power at night. In this configuration, the number of motors was reduced from 14 to ten.

Helios Prototype flying wing moments after takeoff, beginning its first test flight on solar power from the U.S. Navy's Pacific Missile Range Facility on Kauai, Hawaii, July 14, 2001.

On August 13, 2001, the Helios Prototype piloted remotely by Greg Kendall reached an altitude of 96,863 feet (29,524 m), a world record for sustained horizontal flight by a winged aircraft. The altitude reached was more than 11,000 feet (3,400 m) — or more than 2 miles (3.2 km) — above the previous altitude record for sustained flight by a winged aircraft. In addition, the aircraft spent more than 40 minutes above 96,000 feet (29,000 m).

## Crash

On June 26, 2003, the Helios Prototype broke up and fell into the Pacific Ocean about ten miles (16 km) west of the Hawaiian Island Kauai during a remotely piloted systems checkout flight in preparation for an endurance test scheduled for the following month.

On the morning of the accident, weather forecasts indicated that conditions were inside the acceptable envelope, although during the preflight go/no-go review, the weather forecaster gave it a "very marginal GO." One of the primary concerns was a pair of wind shear zones off the island's coast. After a delayed take off, due to the failure of the winds to shift as predicted, Helios spent more time than expected flying through a zone of low-level turbulence on the lee side of Kauai, because it was climbing more slowly than normal, since it had to contend with cloud shadows and the resultant reduction in solar power.

As the aircraft climbed through 2,800 feet (850 m), according to the subsequent mishap investigation report, "At about 30 minutes into the flight, the aircraft encountered turbulence and morphed into an unexpected, persistent, high dihedral configuration. As a result of the persistent high dihedral, the aircraft became unstable in a very divergent pitch mode in which the airspeed excursions from the nominal flight speed about doubled every cycle of the oscillation. The aircraft's design airspeed was subsequently exceeded and the resulting high dynamic pressures caused the wing leading edge secondary structure on the outer wing panels to fail and the solar cells and skin on the upper surface of the wing to rip off. The aircraft impacted the ocean within the confines of the PMRF test range and was destroyed. Most of the vehicle structure was recovered except the hydrogen-air fuel cell pod and two of the ten motors, which sank into the ocean."

In this paper we have taken failure due to persistent, high dihedral configuration and failure due to reduction in solar power with different repair facilities. When the main operative unit fails then warm standby system becomes operative. Failure due to reduction in solar power cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when reduction in solar power. The repair is done on the basis of first fail first repaired.

## 2. ASSUMPTIONS

1.  $\lambda_1, \lambda_2, \lambda_3$  are constant failure rates when failure due to persistent, high dihedral configuration and reduction in solar power respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are  $G_1(t)$ ,  $G_2(t)$  and  $G_3(t)$ ,  $G_4(t)$ .
2. The failure due to reduction in solar power is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to persistent, high dihedral configuration and failure due to sinking caused by reduction in solar power and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous and all random variables are mutually independent.
5. When both the units fail, we give priority to operative unit for repair.
6. Repairs are perfect and failure of a unit is detected immediately and perfectly.
7. The system is down when both the units are non-operative.

### Symbols for states of the System

#### Superscripts O, WS, PDCF, RSPF,

Operative, Warm Standby, failure due to persistent, high dihedral configuration, reduction in solar power respectively

#### Subscripts npdcf, pdcf, rspf, ur, wr, uR

No failure due to persistent, high dihedral configuration, failure due to persistent, high dihedral configuration, failure due to reduction in solar power, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7, 8, 9, 11;

regeneration point – 0, 1, 2, 3, 8, 9, 10

### States of the System

**0( $O_{npdcf}, CS_{npdcf}$ )** One unit is operative and the other unit is warm standby and there is no failure due to persistent, high dihedral configuration of both the units.

**1( $PDCF_{pdcf, urI}, O_{npdcf}$ )** The operating unit failure due to persistent, high dihedral configuration is under repair immediately of Type- I and standby unit starts operating with no failure due to persistent, high dihedral configuration

**2( $RSPF_{rspf, urII}, O_{npdcf}$ )** The operative unit failure due to persistent, high dihedral configuration and reduction in solar power and undergoes repair of type II and the standby unit becomes operative with no failure due to persistent, high dihedral configuration

**3( $RSPF_{rspf, urIII}, O_{npdcf}$ )** The first unit failure due to sinking caused by reduction in solar power and under Type-III multispecialty repairman and the other unit is operative with no failure due to persistent, high dihedral configuration

**4( $PDCF_{pdcf, urI}, PDCF_{pdcf, wrI}$ )** The unit failed due to PDCF resulting from failure due to persistent, high dihedral configuration under repair of Type- I continued from state 1 and the other unit failed due to PDCF resulting from failure due to persistent, high dihedral configuration is waiting for repair of Type-I.

**5( $PDCF_{pdcf, urI}, RSPF_{rspf, wrII}$ )** The unit failed due to PDCF resulting from failure due to persistent, high dihedral configuration is under repair of Type- I continued from state 1 and the other unit failure due to reduction in solar power is waiting for repair of Type-II.

**6( $RSPF_{rspf, urII}, PDCF_{pdcf, wrI}$ )** The operative unit failed due to reduction in solar power is under repair continues from state 2 of Type –II and the other unit failed due to PDCF resulting from failure due to persistent, high dihedral configuration is waiting under repair of Type-I.

**7( $RSPF_{rspf, urII}, PDCF_{pdcf, wrII}$ )** The one unit failure due to reduction in solar power is continued to be under repair of Type II and the other unit failed due to PDCF resulting from failure due to persistent, high dihedral configuration is waiting for repair of Type-II.

**8( $PDCF_{pdcf, urIII}, RSPF_{rspf, wrII}$ )** The one unit failure due to persistent, high dihedral configuration is under multispecialty repair of Type-III and the other unit failure due to reduction in solar power is waiting for repair of Type-II.

**9(PDCF<sub>pdcf,urIII</sub>, RSPF<sub>rspf, urI</sub>)** The one unit failure due to persistent, high dihedral configuration is under multispecialty repair of Type-III and the other unit failure due to reduction in solar power is waiting for repair of Type-I

**10(O<sub>npdof</sub> RSPF<sub>rspf, urIV</sub>)** The one unit is operative with no failure due to persistent, high dihedral configuration and warm standby unit failure due to reduction in solar power and undergoes repair of type IV.

**11(O<sub>npdof</sub> RSPF<sub>rspf, urIV</sub>)** The one unit is operative with no failure due to persistent, high dihedral configuration and warm standby unit failure due to reduction in solar power and repair of type IV continues from state 10.

### Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$$\begin{aligned} p_{01} &= \lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3), p_{02} = \lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3), \\ p_{0,10} &= \lambda_3 / (\lambda_1 + \lambda_2 + \lambda_3), p_{10} = pG_1^*(\lambda_1) + qG_2^*(\lambda_2), \\ p_{14} &= p - pG_1^*(\lambda_1) = p_{11}^{(4)}, p_{15} = q - qG_1^*(\lambda_2) = p_{12}^{(5)}, \\ p_{23} &= pG_2^*(\lambda_1) + qG_2^*(\lambda_2), p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}, \\ p_{27} &= q - qG_2^*(\lambda_2) = p_{28}^{(7)}, \\ p_{30} &= p_{82} = p_{91} = 1, p_{0,10} = pG_4^*(\lambda_1) + qG_4^*(\lambda_2) \\ p_{10,1} &= p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}, p_{10,2} = q - qG_4^*(\lambda_2) = p_{10,2}^{(11)} \end{aligned} \quad (1)$$

We can easily verify that

$$\begin{aligned} p_{01} + p_{02} + p_{03} &= 1, p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1, \\ p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) &= 1, p_{30} = p_{82} = p_{91} = 1 \\ p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) &= 1 \end{aligned} \quad (2)$$

And mean sojourn time is  $\mu_0 = E(T) = \int_0^\infty P[T > t] dt$

### 3. MEAN TIME TO SYSTEM FAILURE

$$\begin{aligned} \emptyset_0(t) &= Q_{01}(t)[s] \emptyset_1(t) + Q_{02}(t)[s] \emptyset_2(t) + Q_{0,10}(t)[s] \emptyset_{10}(t) \\ \emptyset_1(t) &= Q_{10}(t)[s] \emptyset_0(t) + Q_{14}(t) + Q_{15}(t), \emptyset_2(t) = Q_{23}(t)[s] \\ &\emptyset_3(t) + Q_{26}(t) + Q_{27}(t) \\ \emptyset_3(t) &= Q_{30}(t)[s] \emptyset_0(t), \emptyset_{10}(t) = Q_{10,0}(t)[s] \emptyset_{10}(t) + Q_{10,2}(t)[s] \\ &\emptyset_1(t) + Q_{10,2}(t)[s] \emptyset_2(t) \end{aligned} \quad (3-6)$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$$\emptyset_0^*(s) = N_1(s) / D_1(s) \quad (7)$$

where

$$N_1(s) = \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} [Q_{14}^*(s) + Q_{15}^*(s)] + \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} [Q_{26}^*(s) + Q_{27}^*(s)]$$

$$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} Q_{10}^* - \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} Q_{23}^* Q_{30}^* - Q_{0,10}^* Q_{10,0}^*$$

Making use of relations (1) & (2) it can be shown that  $\emptyset_0^*(0) = 1$ , which implies that  $\emptyset_0(t)$  is a proper distribution.

$$\begin{aligned} \text{MTSF} &= E[T] = \frac{d}{ds} \emptyset_0^*(s) \bigg|_{s=0} \\ &= (D_1'(0) - N_1'(0)) / D_1(0) \end{aligned}$$

$$= (\mu_0 + \mu_1 (p_{01} + p_{0,10} p_{10,1}) + (p_{02} + p_{0,10} p_{10,2}) (\mu_2 + \mu_3) + \mu_{10} p_{0,10} / (1 - (p_{01} + p_{0,10} p_{10,1}) p_{10} - (p_{02} + p_{0,10} p_{10,2}) p_{23}) - p_{0,10} p_{10,0})$$

where

$$\begin{aligned} \mu_0 &= \mu_{01} + \mu_{02} + \mu_{0,10}, \mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}, \\ \mu_2 &= \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}, \mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2} \end{aligned}$$

### 4. AVAILABILITY ANALYSIS

Let  $M_i(t)$  be the probability of the system having started from state  $i$  is up at time  $t$  without making any other regenerative state. By probabilistic arguments, we have

$$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t}, M_1(t) = p \overline{G_1}(t) e^{-\lambda_1 t}$$

$$M_2(t) = q \overline{G_2}(t) e^{-\lambda_2 t}, M_3(t) = G_3(t),$$

$$M_{10}(t) = \overline{G_4}(t) e^{-\lambda_3 t}$$

The point wise availability  $A_i(t)$  have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t),$$

$$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]A_8(t) + q_{29}^{(6)}(t)[c]A_9(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t), A_8(t) = q_{82}(t)[c]A_2(t), A_9(t) = q_{91}(t)[c]A_1(t)$$

$$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \quad (8-14)$$

Taking Laplace Transform of eq. (8-14) and solving for  $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (15)$$

where

$$\begin{aligned} N_2(s) &= \{ \hat{q}_{0,10} \hat{M}_{10} + \hat{M}_0 \} [ \{ 1 - \hat{q}_{11}^{(4)} \} \{ 1 - \hat{q}_{28}^{(7)} \} \\ &\hat{q}_{82} \} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} ] + \{ \hat{q}_{01} + \hat{q}_{0,10} \end{aligned}$$

$$\bar{q}_{10,1}^{(11)} \{ \bar{M}_1 \{ 1 - \bar{q}_{28}^{(7)} \bar{q}_{82} \} + \bar{q}_{12}^{(5)} \bar{q}_{23} \bar{M}_3 + \bar{M}_2 \} + \{ \bar{q}_{02} + \bar{q}_{0,10} \bar{q}_{10,2}^{(11)} \} \{ \bar{q}_{23} \bar{M}_3 \} \{ 1 - \bar{q}_{11}^{(4)} \} + \bar{q}_{29}^{(6)} \bar{q}_{91} \bar{M}_1 ]$$

$$D_2(s) = \{ 1 - \bar{q}_{11}^{(4)} \} \{ 1 - \bar{q}_{28}^{(7)} \bar{q}_{82} \} - \bar{q}_{12}^{(5)} \bar{q}_{29}^{(6)}$$

$$\bar{q}_{91} - \{ \bar{q}_{01} + \bar{q}_{0,10} \bar{q}_{10,1}^{(11)} \} [ \bar{q}_{10} \{ 1 - \bar{q}_{28}^{(7)} \bar{q}_{82} \} + \bar{q}_{12}^{(5)} \bar{q}_{23} \bar{q}_{30} ] - \{ \bar{q}_{02} + \bar{q}_{0,10} \bar{q}_{10,2}^{(11)} \} \{ \bar{q}_{23} \bar{q}_{30} \} \{ 1 - \bar{q}_{11}^{(4)} \} + \bar{q}_{29}^{(6)} \bar{q}_{91} \bar{q}_{10} ]$$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] \\ = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2(s)} = \frac{N_2(0)}{D_2'(0)} \quad (16)$$

Where

$$N_2(0) = \{ p_{0,10} \bar{M}_{10}(0) + \bar{M}_0(0) \} \{ [1 - p_{11}^{(4)}] \{ 1 - p_{28}^{(7)} \} - p_{12}^{(5)} p_{29}^{(6)} \} + \{ p_{01} + p_{0,10} p_{10,1}^{(11)} \}$$

$$[ \bar{M}_1(0) \{ 1 - p_{28}^{(7)} \} + p_{12}^{(5)} p_{23} \bar{M}_3(0) + \bar{M}_2(0) ] + \{ p_{02} + p_{0,10} p_{10,2}^{(11)} \} \{ \{ p_{23} \bar{M}_3(0) + \bar{M}_2(0) \} \{ 1 - p_{11}^{(4)} \} + p_{29}^{(6)} \bar{M}_1(0) \}$$

$$D_2'(0) = \mu_0 [ p_{10} (1 - p_{28}^{(7)}) + p_{12}^{(5)} p_{23} ] + \mu_1 [ p_{29}^{(6)} + p_{01} p_{23} - p_{0,10} \{ p_{10,0} \{ 1 - p_{28}^{(7)} \} + p_{23} p_{10,2}^{(11)} p_{23} \} ] + \mu_2 [ (1 - p_{11}^{(4)}) - p_{01} p_{10} - p_{0,10} (p_{10} - p_{10} p_{10,2}^{(11)} + p_{12}^{(5)} p_{10,0}) ] + \mu_3 [ p_{23} p_{12}^{(5)} \{ p_{01} + p_{0,10} p_{10,1}^{(11)} \} + (1 - p_{11}^{(4)}) \{ p_{02} + p_{0,10} p_{10,2}^{(11)} \} ] + \mu_8 [ p_{28}^{(7)} (1 - p_{0,10} p_{10,0} - p_{10} \{ p_{01} + p_{0,10} p_{10,1}^{(11)} \}) ] + \mu_9 [ p_{29}^{(6)} \{ p_{12}^{(5)} (1 - p_{0,10} p_{10,0} - (p_{02} + p_{0,10} p_{10,2}^{(11)})) \} ] + \mu_{10} [ p_{29}^{(6)} \{ p_{12}^{(5)} (1 - p_{0,10} p_{10,0} + (p_{02} + p_{0,10} p_{10,2}^{(11)})) \} ]$$

$$\text{and } \mu_3 = \mu_{30}, \mu_9 = \mu_{91}, \mu_8 = \mu_{81}$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^t A_0(z) dz \quad \text{So that } \bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \quad (17)$$

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t)$$

$$\text{So that } \bar{\lambda}_d(s) = \frac{1}{s} - \bar{\lambda}_u(s) \quad (18)$$

Similarly, we can find out

1. The expected busy period of the server when there is failure due to persistent, high dihedral configuration and reduction in solar power in (0,t]-R<sub>0</sub>

2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H<sub>0</sub>

3. The expected number of visits by the multispecialty repairman Type-III, Type-IV for repairing the identical units in (0,t]-W<sub>0</sub>, Y<sub>0</sub> respectively

## 5. BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to persistent, high dihedral configuration and reduction in solar power, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

$$C = \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) = K_1 A_0 - K_2 R_0 - K_3 H_0 - K_4 W_0 - K_5 Y_0$$

where

K<sub>1</sub> - revenue per unit up-time, K<sub>2</sub> - cost per unit time for which the system is busy under repairing, K<sub>3</sub> - cost per visit by the repairman type- I or type- II for units repair,

K<sub>4</sub> - cost per visit by the multispecialty repairman Type- III for units repair

K<sub>5</sub> - cost per visit by the multispecialty repairman Type- IV for units repair

## 6. CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to persistent, high dihedral configuration and reduction in solar power increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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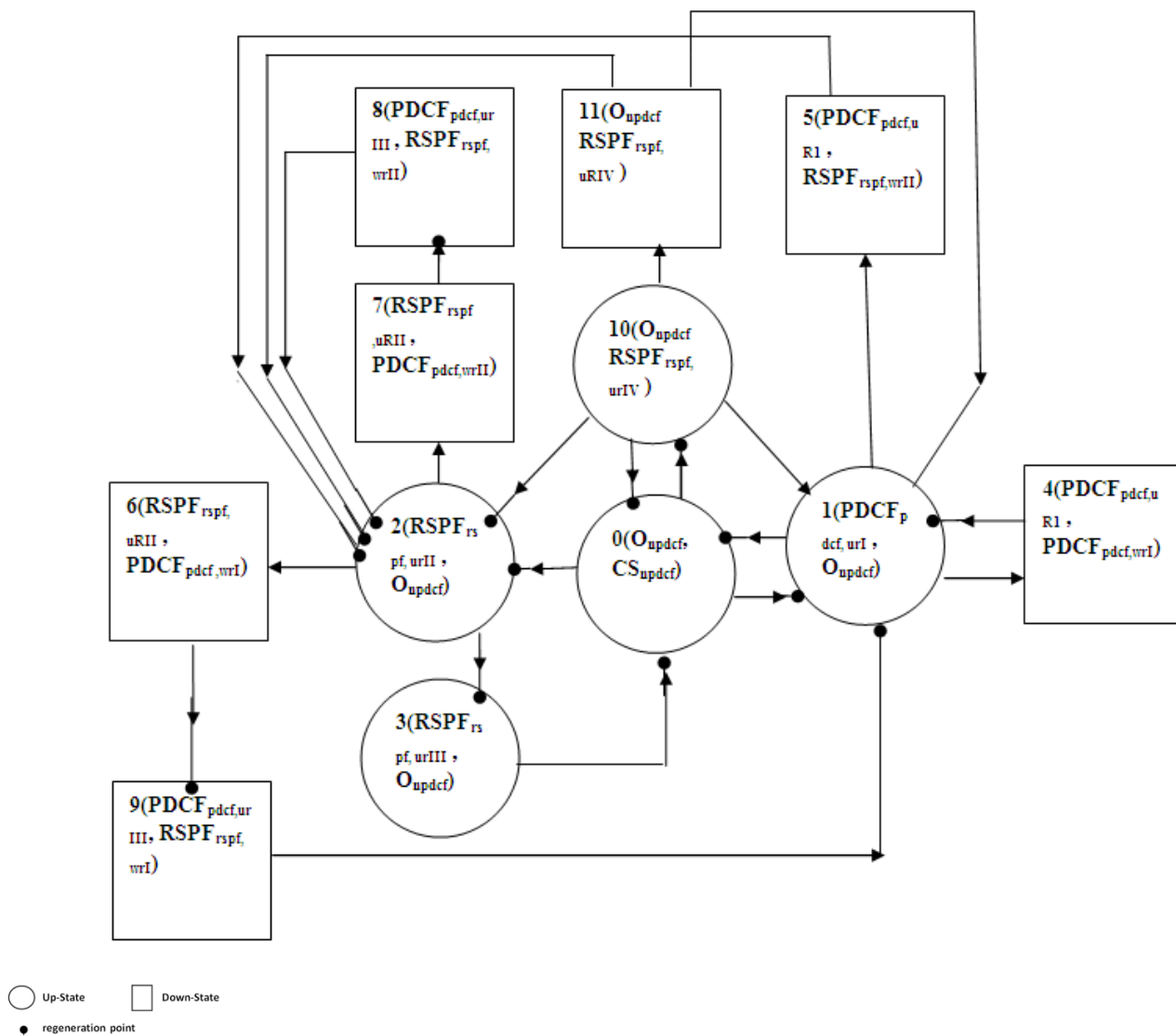


Fig. 1: The State Transition Diagram